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[1]**Arinov E.,** [2]**Kudaikulov A. K.,** [3]**Tashev A. A.**
[1]Zhezkazgan University named after O. A. Baikonurov, Zhezkazgan, RK
[2,3]Institute of Information and Computing Technologies of the KN MES RK, Almaty, RK

[1]*E-mail: arinov91@mail.ru*
[2]*E-mail: kudaykulov2006@mail.ru*
[3]*E-mail: Azattash@mail.ru*

AN ANALYTICAL SOLUTION TO THE PROBLEM OF THE THERMOMECHANICAL STATE OF A ROD OF LIMITED LENGTH, WITH SIMULTANEOUS PRESENCE OF END TEMPERATURES AND LATERAL HEAT EXCHANGE

БҮЙІР ЖЫЛУ АЛМАСУ ӘСЕРІНДЕГІ ӨЗЕКТІҢ ТЕРМОМЕХАНИКАЛЫҚ КҮЙІН ЖӘНЕ ҰШТАРЫНДАҒЫ ЖЕРГІЛІКТІ ТЕМПЕРАТУРАНЫ ЗЕРТТЕУ

ИССЛЕДОВАНИЕ ТЕРМОМЕХАНИЧЕСКОГО СОСТОЯНИЯ СТЕРЖНЯ НАХОДЯЩЕЙСЯ ПОД ВОЗДЕЙСТВИЕМ БОКОВОГО ТЕПЛООБМЕНА И ЛОКАЛЬНЫХ ТЕМПЕРАТУР НА КОНЦАХ

Abstract. This article deals with the problems of numerical study of the thermomechanical state of rods. On the basis of the fundamental law on the change in the amount of heat, an equation of the established thermal conductivity for a horizontal rod of limited length and a constant cross section is constructed through a fixed cross-section in a time $\partial\tau$. In this case, different temperatures are set at the two ends of the investigated rod, and heat exchange with the surrounding medium takes place through the lateral surface. In addition, the investigated rod is made of thermal protective material ANV-300. The determining law of the distribution of temperature, of all the corresponding deformations and stresses, and also of the displacement along the length of the investigated rod. The values of the thermal elongation and the resulting axial force are calculated.

In a complex thermal zone, bearing components of reactive and hydrogen engines, nuclear and thermal power stations, processing lines of processing industries, as well as internal combustion engines operate. The reliable operation of these structures will depend on the conditions of the thermoelectric power of the bearing components. Therefore, this study is devoted to a numerical study of the state of the thermoelectric power of the structural components in the form of rods of limited length, bounded at both ends.

The proposed computational algorithm is based on the principle of energy conservation. In this case, all types of integrals in the functional energy formulas are integrated analytically. In this case, the numerical solutions obtained will have high accuracy.

Keywords: the temperature, the rod, the thermal energy, the algorithm.

Аңдатпа. Жылу мөлшерінің өзгеруінің негізгі заңына сүйене отырып, тұрақты коэффициенті бар қарапайым дифференциалдық теңдеу салынды, ол зерттелетін шектеулі ұзындықтағы шыбықтың ұзындығы бойымен температураны бөлу өрісін сипаттайды. Бүйір бетінің ауданы бойынша оны қоршаған ортамен жылу алмасу жүреді. Өзектің екі ұшында әртүрлі жергілікті температура берілген.

Анықталған: температураның таралу өрісі, шыбықтың ұзару шамасы, пайда болатын осьтік күш шамасы, деформация мен кернеудің барлық компоненттерінің таралу заңдылықтары, серпімді қозғалыс компонентінің таралу өрісі.

Ұсынылған есептеу алгоритмі энергияны сақтау принципіне негізделген. Бұл жағдайда функционалды энергия формулаларындағы интегралдардың барлық түрлері аналитикалық түрде біріктіріледі. Бұл жағдайда алынған сандық шешімдер жоғары дәлдікке ие болады.

Түйін сөздер: жылу мөлшері, жылу өткізгіштік, жылу алмасу коэффициенті, жылу кеңеюі, серпімділік модулі.

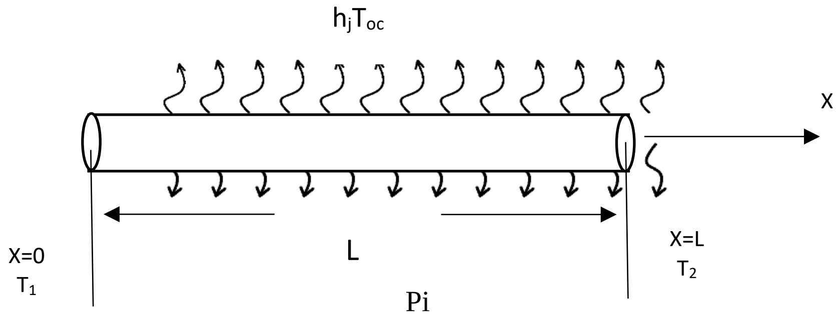
Аннотация. На основе фундаментального закона изменения количества тепла построено обыкновенное дифференциальное уравнение с постоянным коэффициентом, которое описывает поле распределения температуру по длине исследуемого стержня ограниченной длины. По площади боковой поверхности которого происходит теплообмен с окружающий ее средой. На двух концах стержня даны разные локальные температуры.

Определены: поле распределения температуры, величина удлинения стержня, величина возникающего осевого усилия, законы распределения всех составляющих деформации и напряжении, поле распределения упругой составляющей перемещения.

Предлагаемый вычислительный алгоритм основан на принципе сохранения энергии. При этом все типы интегралов в функциональных формулах энергии интегрируются аналитически. При этом полученные численные решения будут иметь высокую точность.

Ключевые слова: количество тепла, теплопроводность, коэффицинет теплообмена, теплового расширения, модуль упругости.

We consider a horizontal rod of limited length and a constant crossed section whose area $F(cm^2)$. He axis ox of the rod is directed from the left to the right which coincides with the axis of the rod. At the left end of the rod, the temperature$T_1[c^0]$, is given, and the direction $T_2[c^0]$. In this case $T_1 > T_2$. Through the lateral surface of the rod, heat exchange takes place with its surrounding medium. In this case, the heat transfer coefficient $h\left[\frac{watt}{cm^2 \cdot c^0}\right]$, and the ambient temperature $T_{oc}[c^0]$. The calculation scheme of the process is shown in Fig. 1



Cture 1. The calculation scheme of the problem

It is required to determine:

1) The law of temperature distribution along the length of the investigated rod.
2) Determine the amount of thermal elongation of the test rod.

In case of pinching the two ends of the rod, it is necessary to determine:

3) The arising axial forces.

4) The field of distribution of the components of deformations and stresses.

5) The field of distribution of displacement.

The physical and mechanical properties of the material of the rod under investigation are characterized by the coefficients of thermal conductivity $K_{xx}\left[\frac{watt}{cm^2 \cdot c^0}\right]$, thermal expansion $\alpha\left[\frac{1}{c^0}\right]$ and elastic modulus $E\left[\frac{kg}{cm^2}\right]$. If we take into account that the investigated process of the rod material is much larger than the cross-sectional area, then it is possible to neglect the temperature gradients in the directions perpendicular to the axis of the rod without significant error, and take the temperature constant at each point of the cross section perpendicular to the axis . With this assumption, a temperature with a function of only one independent variable x, and the field of temperature distribution along the length of the rod can be described by an ordinary differential equation.

According to the fundamental law of thermophysics, the amount of heat passing through the time dt through the cross sections of the rod at a distance of x [cm] from its left end will be

$$-K_{xx}F\frac{dT}{dx}d\tau \qquad (1)$$

where $T(x)-$ is the temperature distribution field, which is still unknown.

At that time, the amount of heat passing through the time dt through the cross section, located at a distance $x+dx$[см] from the left end of the rod, will be equal to

$$-K_{xx}F\left(\frac{dT}{dx}+\frac{d^2T}{dx^2}dx\right)d\tau \qquad (2)$$

In addition, the portion of the rod enclosed between the sections spaced from the left end of the rod at a distance of x and $x+dx[cm]$, due to the thermal conductivity process, acquires during the time dt the amount of heat equal to the difference of the indicated quantities (1) and (2) e.

In addition, the portion of the rod enclosed between the sections spaced from the left end of the rod at a distance of x and $x+dx$[cm], following the heat conduction process, acquires in the time dt the amount of heat equal to the difference of the indicated amounts (1) and (2),

$$K_{xx}F\frac{d^2T}{dx^2}d\tau \qquad 3)$$

It should also be noted that during this same time, a heat loss equal to

$$hPdx(T-T_{\text{oc}})d\tau \qquad (4)$$

where $P[cm]$ is the cross sectional.

But since the process we are investigating is steady-state, i.e. stationary, then from (3-4) we have

$$K_{xx}F\frac{d^2T}{dx^2}dxd\tau = hPdx(T-T_{\text{oc}})d\tau \qquad (5)$$

From this, for the problem under consideration, we determine the equation for the steady-state heat conductivity

$$\frac{d^2T}{dx^2}=\frac{hP(T-T_{\text{oc}})}{K_{xx}F} \qquad (6)$$

For convenience, we introduce the notation

$$a^2=\frac{hP}{K_{xx}F} \qquad 7)$$

considering that the ambient temperature $T_{\text{oc}}=const, 0\le x\le l$, then we have

$$\frac{d(T-T_{0c})}{dx}=\frac{dt}{dx} \qquad (8)$$

hence we also obtain $\frac{d^2T}{dx^2}=\frac{d^2(T-T_{oc})}{dx^2}, 0\le x\le l$ 9)

Taking (7) and (9) into account, we rewrite (6)

$$\frac{d^2(T-T_{oc})}{dx^2}-a^2(T-T_{oc})=0 \quad (10)$$

This equation is an ordinary differential equation with constant coefficients. Then its general integral will be

$$T-T_{oc}=C_1e^{ax}+C_2e^{-ax}, \quad 0\le x\le l$$

(11)

where C_1 and C_2 are constants of integration. Their values are determined from the boundary conditions at the ends of the rod.

$$(x=0)=T_1[c^0];T(x=l)=T_2[c^0]; \quad (12)$$

$$\left.\begin{aligned}T_1-T_{oc}&=C_1+C_2\\ T_2-T_{oc}&=C_1e^{al}+C_2e^{-al}\end{aligned}\right\} \quad (13)$$

From these systems, the values C_1 *and* C_2.

$$\left.\begin{aligned}C_1&=\frac{(T_2-T_{oc})-(T_1-T_{oc})e^{-al}}{2sh(al)}\\ C_2&=\frac{(T_1-T_{oc})e^{al}-(T_2-T_{oc})}{2sh(al)}\end{aligned}\right\} \quad (14)$$

Substituting (14) into (11), we determine the field of temperature distribution along the length of the rod under consideration, taking into account the operating conditions [2]

$$T(x,h,K_{xx},P,F,T_{oc})=T_{oc}+\frac{(T_2-T_{oc})sh(ax)+(T_1-T_{oc})sha(l-x)}{sh(al)}\ ,0\le x\le l \quad (15)$$

On the basis of the fundamental theory of thermal physics, it is possible to determine the elongation of the rod under consideration if it is pinched by one end and the other is free

$$\Delta l_T=\int_0^l \alpha T(x)dx=\alpha\int_0^l T(x)dx=\alpha\{T_{oc}l+[(T_2-T_{oc})(ch(al)-1)/a-(T_1-T_{oc})(1-ch(al)/a)]/sh(al)\} \quad (16)$$

In the event that both ends of the rod are clamped, an axial compressive force R is produced in it, which will be directed along its axis ox. Its value is determined by the corresponding Hooke law [3]

$$R=-\frac{\Delta l_T EF}{l}=-\frac{\alpha EF}{l}\{T_{oc}l+[(T_2-T_{oc})(ch(al)-1)/a-(T_1-T_{oc})(1-ch(al)/a)]/sh(al)\} \quad (17)$$

In this case, according to the length of the investigated rod, the distribution law of the thermoelastic component of the voltage t can be determined according to the generalized Hooke's law

$$\sigma=\frac{R}{F}=-\frac{\alpha E}{l}\{T_{oc}l+[(T_2-T_{oc})(ch(al)-1)/a-(T_1-T_{oc})(1-ch(al)/a)]/sh(al)\} \quad (18)$$

Then the distribution law of the corresponding thermo-elastic component of the deformation is also determined according to Hooke's law

$$\varepsilon=\frac{\sigma}{E}=-\frac{\alpha}{l}\{T_{oc}l+[(T_2-T_{oc})(ch(al)-1)/a-(T_1-T_{oc})(1-ch(al)/a)]/sh(al)\} \quad (19)$$

Further, according to the theory of thermal physics, the law of distribution of the temperature component of deformation

$$\varepsilon_T(x) = -\alpha T(x) = -\alpha\left\{T_{oc} + \frac{(T_2 - T_{oc})sh(ax) + (T_1 - T_{oc})sha(l-x)}{sh(al)}\right\}, 0 \le x \le l \qquad (20)$$

Then the temperature component of the voltage is already determined according to Hooke's law

$$\sigma_T(x) = E\varepsilon_T(x) = -\alpha E\left\{T_{oc} + \frac{(T_2 - T_{oc})sh(ax) + (T_1 - T_{oc})sha(l-x)}{sh(al)}\right\}, 0 \le x \le l \qquad (21)$$

After this, according to the theory of thermo elasticity, it is possible to determine the law of distribution of the elastic component of deformation

$$\varepsilon_x(x) = \varepsilon - \varepsilon_T(x) = -\frac{\alpha}{l}\left\{T_{oc}l + \left[(T_2 - T_{oc})(ch(al) - 1)/a - (T_1 - T_{oc})(1 - ch(al)/a)\right]/sh(al)\right\} +$$
$$\alpha\left\{T_{oc} + \frac{(T_2 - T_{oc})sh(ax) + (T_1 - T_{oc})sha(l-x)}{sh(al)}\right\}, 0 \le x \le l \qquad (22)$$

Then, according to Hooke's law, we can determine the law of distribution of the elastic component of the voltage

$$\sigma_x(x) = E\varepsilon_x(x) = \sigma - \sigma_T(x) = -\frac{\alpha E}{l}\left\{T_{oc}l + \left[(T_2 - T_{oc})(ch(al) - 1)/a - (T_1 - T_{oc})(1 - ch(al)/a)\right]/sh(al)\right\} +$$
$$\alpha E\left\{T_{oc} + \frac{(T_2 - T_{oc})sh(ax) + (T_1 - T_{oc})sha(l-x)}{sh(al)}\right\}, 0 \le x \le l$$

23)

Finally, we can determine the law of distribution of the displacement of the cross-section of the rod. It is determined from the Cauchy relations

$$\varepsilon_x(x) = \frac{\partial u}{\partial x}; \Rightarrow U = \int \varepsilon_x(x)dx + C \qquad (24)$$

Here the value of the constant C is determined from the pinning conditions *U(x=0)=0*. Then we have

$$U(x) = -\alpha\left[T_{oc} + \frac{chal - 1}{alshal}(T_1 + T_2 - 2T_{oc})\right]x + \alpha\left\{T_{oc}x + \frac{1}{ashal}[(T_2 - T_{oc})chax - (T_1 - T_{oc})]\right\}$$
$$+ \frac{\alpha}{ashal}\left[(T_1 - T_{oc})chal - (T_2 - T_{oc})\right] \qquad 25)$$

Then we have l=100см, $K_{xx} = 100\frac{\text{Вт}}{\text{см с}^0}$; h=10$\frac{\text{Вт}}{\text{см}^2\text{с}^0}$; $T_{\text{ос}} = 20^0$C; α =125·10^{-7} $\frac{1}{\text{с}^0}$; E=2·10$^6\frac{kg}{cm^2}$; T_1=600^{0}C; T_2=100^{0}C; r=1cm.

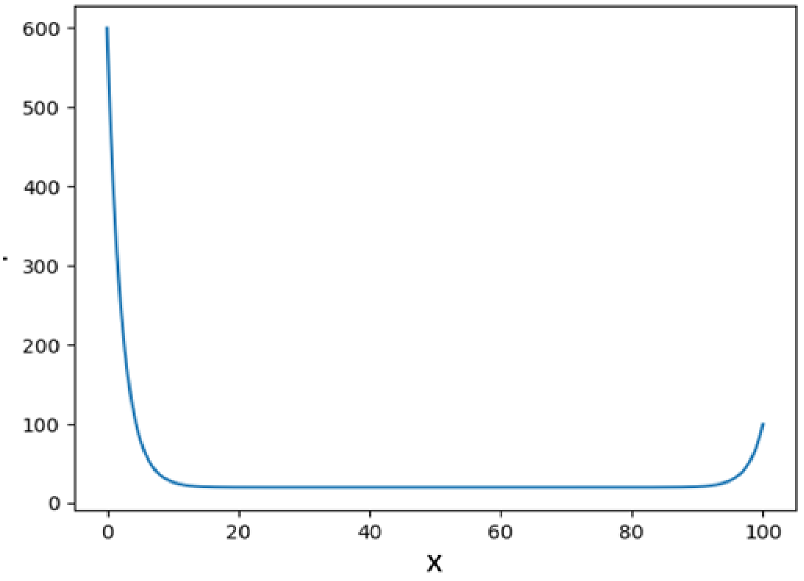
Then we get the results shown in Figure-2. In Figure-2, a) the law of the distribution of temperature along the length of the rod is given. The resulting law of distribution of deformation components is given in Figure-2, b). It can be seen from the figure that the thermo-elastic component of the deformation Ɛ-is constant along the entire length of the rod.

At that time, the elastic component of the deformation $\mathcal{E}_x(x)$, on stretches near the jamming, has a stretching character. In the middle section of the rod, $\mathcal{E}_x(x)$, has a compressive character. The temperature component of the deformation $\mathcal{E}_T(x)$ along the entire length has a compressive character. Its maximum value corresponds to the highest temperature.

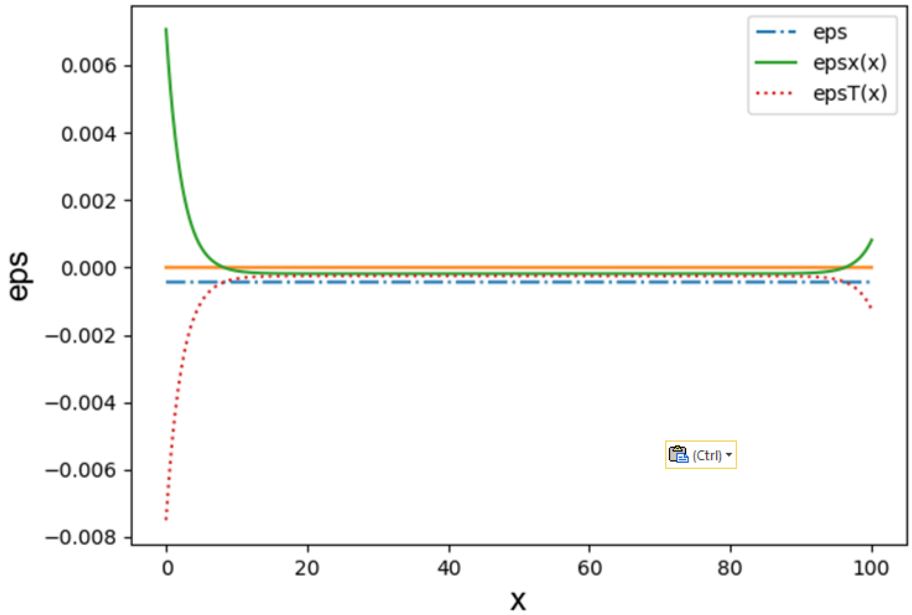
The nature of the component stresses is similar to the corresponding deformations. This is clearly seen from Figure-2, c). In Figure-2, d) the distribution field for the displacement of the cross-sections of the rod is given. It can be seen from the figure that the cross-sections of the rod in section $0 < x \leq 6{,}9$ are moving in the direction of the x axis. At that time, the largest displacement Umax1 = 0.0043092 cm corresponds to the coordinate cross-section of which x = 8 cm;

The cross sections of the rod located in the section 70 <x <l = 100 cm move against the direction of the axis ox. Here, the largest displacement U_{max2}*=-0,0016472* cm corresponds to a cross section whose coordinate is x = 94 cm. Moreover, $/U_{max1}/ / /U_{max2}/ = 2{,}61639$;

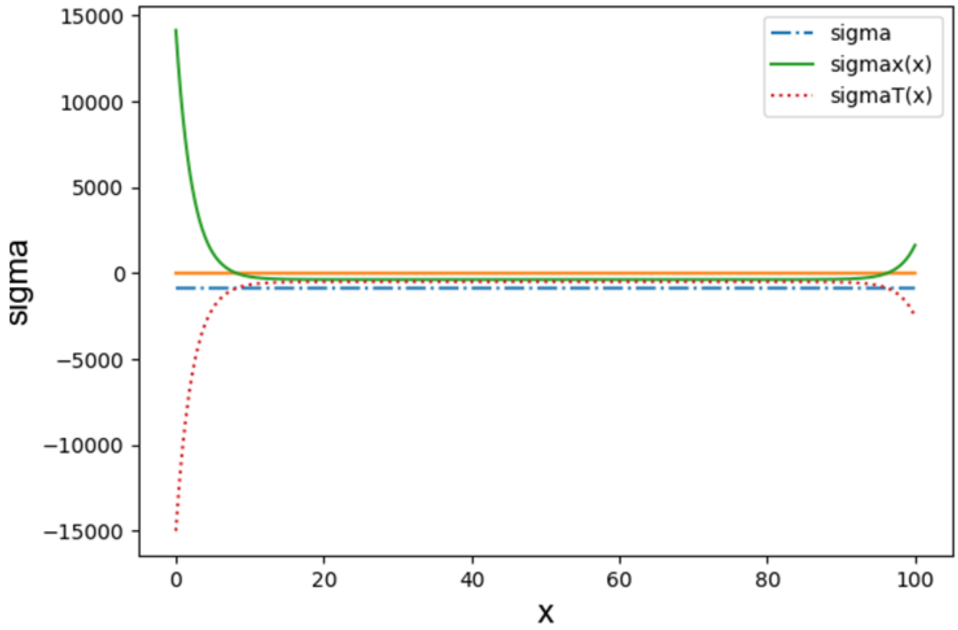
a)The temperature



) the deformation



c) voltage



d) move

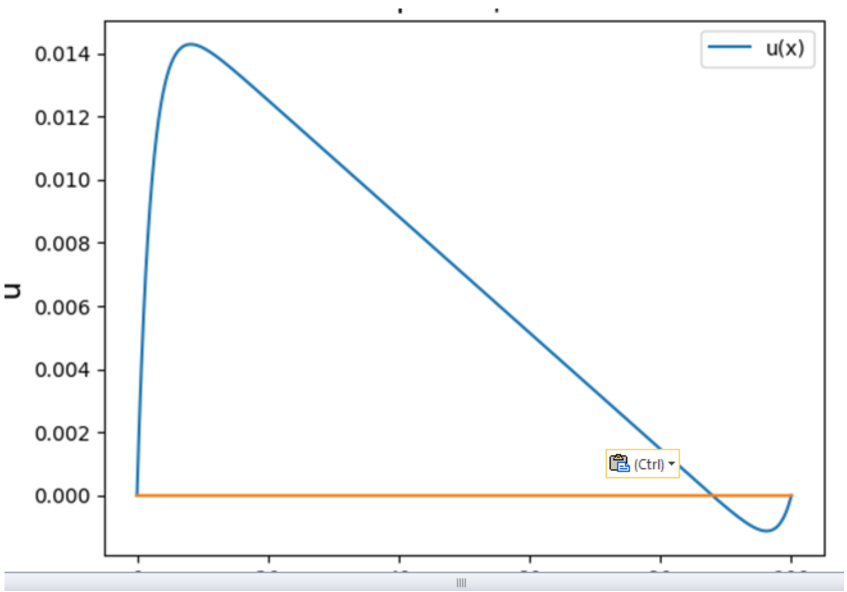


Figure - 2. The laws of distribution of temperatures, strains, stresses and displacements

References

1. Kudaykulov AK, Kenzhegul BZ, Myrzasheva AN Mathematical model of the steady field of temperature distribution along the length of the rod of limited length in the presence of local temperature, heat flow, heat transfer and heat insulation. Science and new technologies, №5, Bishkek, 2009, 17-2. (in Russ.).

2. Kudaykulov AK, Tuleuova R. Amirtaev KB, Tokkuliev BM, "Steady napryazhenno deformed state zhestko-zakreplennogo two ends partially insulated rod in the presence of heat flow, heat transfer and temperature," Proceedings Fifth All-Russian Scientific Conference with international participation (29-31 May 2008). Part 1 Mathematical models of mechanics, strength and reliability of structural elements, Mat. modeling and edges. tasks SamGTU, Samara, 2008, 161-164. (in Russ.).

3. AK Kudaykulov, Mathematical (finite element) modeling of applied problems of heat distribution in one-dimensional structural elements. - Turkestan: Baiterek - 2009. - 168 p.

4. Kenzhegul BZ, Kudaykulov AK, AN Myrzasheva Numerical study of the extension rod superalloy based on the availability of all types of sources. Proceedings of the universities. - Bishkek, 2009. - №4. -3-7. (in Russ.).

5. Tashenova JM, Nurlybaeva EN, Zhumadillaeva AK, AK Kudaykulov The computational algorithm and simulation thermostressed state bar of heat-resistant alloy with heat exchange, thermal insulation and temperature constant intensity. Basic research. - 2012. - № 3-3. - P. 660-664. (in Russ.).

6. AS Ivanov The mathematical analogy in continuum mechanics. Monograph. Moscow, Moscow State Open University, 2009 180. (in Russ.).

7. X Gu, X Dong, M Liu, Y Wang - Heat Transfer-Asian Research, 2012 - Wiley Online Library.

8. Aytaliev Sh.M., Kudaykulov AK Mardonov B. Mechanics sticking bruilnyh columns in oil and gas wells. Atyrau-Almaty: Publishing "Evreux", 1999, -82. (in Russ.).

9. Chernyaeva T. P. and Ostapov A. V., Problems of Atomic Science and Technology. Ser. Physics of Radiation Effect and Radiation Material Science, (87) 5, 16 (2013). (in Eng.).

10. Zelensky V. F., Problems of Atomic Science and Technology. Ser. Nuclear Physics Investigations (85) 3, 76 (2013). (in Eng.).

11. M.L.F. Lerch, M. Petasecca, A. Cullen et al., Radiation Measurements 46, 1560 (2011). Gestrin SG Localization of Frenkel excitons on dislocations. Gestrin, A.N. Salnikov. News of universities. Physics. 2005. № 7. P. 23-25. (in Eng.).

12. Bezshyyko A., Vyshnevskyi I.M., Denisenko R.V. et al., Nucl. Phys. At. Energy 12, No. 4, 400 (2011). (in Eng.).

13. Gestrin SG Localization of Frenkel excitons on dislocations. Gestrin, A.N. Salnikov. News of universities. Physics. 2005. № 7. P. 23-25. (in Eng.). (in Eng.).

14. Tungatarov A., D.K. Akhmed-Zaki. Cauchy problem for one class of ordinary differential equations// Int. J. of Mathematical Analyses. 2012, vol.6, no 14, 695-699. (in Eng.).

15. Meirmanov A., Mathematical models for poroelastic flows, Atlantis Press// Paris, 2013,478 pp. (in Eng.).

16. Kulpeshov B.Sh., Macpherson H.D., Minimality conditions on circularly ordered structures. Mathematical Logic Quarterly, 51 (2005), pp. 377-399. (in Eng.).

17. Kulpeshov B.Sh., On $\aleph_0$-categorical weakly circularly minimal structures. Mathematical Logic Quarterly, volume 52, issue 6, 2006, pp. 555-574. (in Eng.).

18. Yerofeyev VL, Semenov PD Heat. - M.: ICC Akademkniga.-2006-488. (in Russ.).

19. VN Lukanin Teplotehnika. M .: Higher shkola.-2002-671. (in Russ.).

20. Nozdrev V.F. Course of thermodynamics. - Moscow: Mir, 1967. - 247 p. (in Russ.).